Beyond Low-Gribov theorem for high energy interactions of scalar and gauge particles.*

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Abstract

We obtain a generalization of the Low theorem for non-Abelian boson emission in collision of scalar and gauge vector particles and its extension to high energy collisions for small transverse momenta of produced particles. We demonstrate that in the case of particles with spin the direct extension the Low formula to high energy is in contradiction with the correct amplitude factorization behavior. Consideration of different kinematical regions and use of methods of dual models allows us to separate contributions of intermediate excited states and standard spin corrections in the Low formulae. We show that the amplitude factorization occurs at high energy due to the contribution of the intermediate states which is additional to the gluon production amplitude for the scalar particle collision.

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1 Introduction.

In recent years there has been significant interest to the experimental and theoretical studies of deep inelastic scattering [1] when provided with very useful information concerning the further development of high-energy physics. In turn, the Low theorem [2] gives us low energy static characteristics like charges, magnetic moments and others. As well known, one can extend its applicability to the region of high-energy collisions [3]. In QED, it proves possible to calculate the inelastic amplitude for the photon production with a small transverse momentum in terms of the elastic amplitude and its derivatives. The next steps were to consider the non-Abelian generalization and to take into account the contribution of excited intermediate states in the corresponding channels (in the channels $s_1, s_{1'}, s_2$, and $s_{2'}$ in Fig.1 where $s_1 = -2(p_1 k),\ s_{1'} = 2\,(p_4 k)\,,\ s_2 = -2(p_2 k)$ and $s_{2'} = 2\,(p_3 k)$). These problems have been considered by Lipatov for scalar case both in the multi-Regge kinematics [4] ($s \gg s_i \gg m_{ch}^2$, where a m_{ch} is the characteristic mass scale of elementary elastic process in Fig.1) and in the fragmentation region [5] ($s \gg s_2 \sim s_{2'} \sim m_{ch}^2$). The gauge massless particles (gluons or gravitons) are radiated in collisions of scalar particles. There the inelastic amplitude can be considered as the dispersion representation in the variables $s_1, s_{1'}, s_2$, and $s_{2'}$. The Low formula can be considered as the contribution of the ground scalar states in this dispersion relation as pole in $s_i = 0$. The contributions of the excited intermediate states is represented by the dispersion integral with subtraction in $s_i = 0$. Such the representation allows the low and high energy regions to be consistent . At low energy the contribution of intermediate states dies out and we have the Low formula. At high energy the contribution of the intermediate states is necessary for the correct analytical amplitude behavior. The general condition of analyticity of the inelastic amplitude in the $s_1, s_{1'}, s_2$ and $s_{2'}$ fixes the contribution of excited intermediate states up to an additive constant which provide the subtraction of the dispersion integral in $s_i = 0$. The constant depends on a chosen model and may depend on a sort of external on-shell particles in the given channel. The value of the constant in Ref. [4, 5] was estimated in terms of the open and closed string models. It equals $\Gamma'(1) = -c_E$ for the open string and $2\log 2 - c_E$ for the closed one where c_E is the Euler constant. In this case the gluon or the graviton plays a role of the gauge particle and tachyons correspond to on-shell scalar particles. The next step was to consider scattering of gauge

particles where the gluon and the graviton were considered as the next excited states for open and closed strings. Such an inelastic amplitude for five gauge massless particles has been calculated in Ref. [6] in the multi-Regge kinematics. At first sight, it seems that the additive constant is the same as for the tachyon collision because of the amplitude factorization, but in fact there exist some additional terms which might contribute to it. In this kinematics we do not have any obvious idea how to single out the contribution of excited intermediate states to clarify the situation. It is necessary to consider another kinematical region, a fragmentation region.

The reason to deal with the amplitude including gauge and scalar particles is to compare contributions of the bremsstrahlung radiation from scalar and vector particle and to find the dependence of the additive constant on the nature of on-shell external particles. Moreover, we would like to investigate the transformation of every contribution in detail, using the string case as a guide, and going from small s_i (the Low formulae are valid) through the fragmentation region $s_i \sim m_{ch}^2 \sim \frac{1}{\alpha'}$ (where α' is a slope of Regge trajectory and the contribution of intermediate particles is yet to be controlled) to large $s_i \gg m_{ch}^2 \sim \frac{1}{\alpha'}$ (multi-Regge kinematics).

This paper is also concerned with the contribution of gauge non-Abelian particles to the Low theorem in the field theory. We demonstrate that in the case of particles with spin the direct extension the Low formula to high energy is in contradiction with the correct amplitude factorization behavior. Comparison of its continuation to the Regge kinematics (where the perturbative theory is valid) with string expressions provides with an additional test (in particular, in the sense of limit $\alpha' \to 0$ [12]) and shows that it is necessary to take into account the contribution of intermediate particles additional to the scalar scattering case. It is the use of the bosonic string theory which allows us to control all contributions in different kinematical regions. In doing so, in contrast with loop calculations [13], Born approximation appears to be enough.

Section (2) deals with the Low theorem for scalar and vector Yang-Mills particles. We extend it to different kinematics in agreement with Gribov's requirement and demonstrate that some corrections are in contradiction with correct amplitude factorization. In section (3) we consider the same process in the string theory in the multi-Regge kinematics where we take into account the excited intermediate states and obtain the amplitude with the correct factorization. The fragmentation region is considered in section (4), which

allows us to separate the contribution of the all intermediate excited states and ground states and to consider the all kinematical regions. As a result we observed that the restoration of the amplitude factorization is due to a new additional type of the intermediate excited states.

2 Low theorem in the case of Yang-Mills scalar and vector particles.

The Low theorem, owing to gauge invariance, allows us to obtain a soft vector particle production amplitude in terms of the elastic one and its derivatives. Our presentation follows [7, 8]. Now we are going to generalize this procedure for a non-Abelian field theory. In this case the Lagrangian is given by

$$\begin{aligned} L = -\tfrac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^a_{bc} A^b_\mu A^c_\nu\right)^2 \\ + \left[\left(\partial_\mu - g A_{\mu\ a} T^a\right)\Phi\right]^+ \left[\left(\partial^\mu - g A^\mu_a T^a\right)\Phi\right] + \cdots \ , \end{aligned} \tag{1}$$

where T^a are generators of $SU(N)$ gauge group in the adjoint representation in which their matrix elements are given as structure constants of this group

$$\begin{aligned} \left(T^a\right)_{bc} = f^a_{bc} = f^{ab}_c = f_{abc} \\ \left[T^a, T^b\right] = i f^{ab}_c T^c \end{aligned} \quad . \tag{2}$$

The basic elastic amplitude with one vector and three scalar particles (see Fig.3) will be denoted $l_{2\ \nu} A^{\nu,\ a_1,a_2,a_3,a_4}(p_1,p_2,p_3,p_4)$ where $l_{2\ \nu}$ is the polarization vector of the gauge particle, a_i are color indices of the gauge group and p_i are the corresponding momenta of particles. The inelastic amplitude is represented by (see Fig.4)

$$A_g(l,k,\ldots) = A^{ext}_g(l,k,\ldots) + A^{int}_g(k,l,\ldots) \ .$$

where l and k are the polarization vector and the momentum of the radiated gluon with the color index a respectively. A^{int}_g is the contribution of diagrams in which an gluon is radiated from an internal line. The sum of pole diagrams such that the additional gluon is radiated from an external line is given in

the form

$$A_g^{ext\ a,\ a_1,a_2,a_3,a_4}\left(l,k;p_1;l_2,p_2;p_3;p_4\right)=$$

$$=ig\sum_c f_c^{a\,a_2}\frac{(l_{2\ \nu}(lp_2)-l_{\ \nu}(l_2k)+(l_2l)k_{\ \nu})}{kp_2}A^{\nu,\ a_1,c,a,a_5}\left(\ldots,p_2+k,\ldots\right)+$$
$$+ig\sum_{i\neq 2}\sum_{c_i} f_{c_i}^{a\,a_i}\frac{(lp_i)}{kp_i}\left(l_{2\ \nu}A^{\nu,\ \ldots,c_i,\ldots}\left(\ldots,p_i+k,\ldots\right)\right) \tag{3}$$

All particles are incoming, so that $\sum_i p_i=-k$. Using the gauge invariance $l\rightarrow l+ck$ we note that

$$A_g^{ext\ a,\ a_1,a_2,a_3,,a_4}\left(l=k,k;\ldots;\ldots\right)=$$
$$=ig\sum_i\sum_{c_i} f_{c_i}^{a\,a_i}\left(l_{2\ \nu}A^{\nu,\ \ldots,c_i,\ldots}\left(\ldots,p_i+k,\ldots\right)\right) \tag{4}$$

Following Ref.[7], we now define more carefully a procedure for the extrapolation from elastic $A^{\nu\ \ldots,c_i,\ldots}\left(\ldots,p_i',\ldots\right)$ with a physically realizable set of momenta $\{p_i'\}$ such that $\sum_i p_i'=0$ and $p_i'^{\,2}=m_i^2$ to $A^{\nu\ \ldots,c_i,\ldots}\left(\ldots,p_i,\ldots\right)$ of the form given in Eq.(3), where one momentum is unphysical and the rest are the same as in A_g. Let $p_i'=p_i-\xi_i\left(k\right)$, then the vectors ξ_i must have the properties $\sum_i\xi_i\left(k\right)=-k,(p_i,\xi)=0,\xi_i\left(0\right)=0,\left(\frac{\partial\xi_i}{\partial k_{\ \mu}}\right)_{k_{\ \mu}=0}=c<\infty$. It is possible for the ξ's to be defined so that scalar variables are the same to first order in k whether expressed in terms of p_i or p_i'.

Now we can derive the relation between the elastic amplitude with on-mass-shell momenta and the elastic amplitude with one unphysical momentum

$$A^{\nu,\ \ldots,c_i,\ldots}\left(\ldots,p_i+k_{,\ldots}\right)=A^{\nu,\ \ldots,c_i,\ldots}\left(\ldots,p'_{i,\ldots}\right)+$$
$$\sum_j\xi_j\frac{\partial}{\partial p_j}A^{\nu,\ \ldots,c_i,\ldots}\left(\ldots,p_{i,\ldots}\right)|_{\{p_i\}=\left\{p_i'\right\}}+k\frac{\partial}{\partial p_i}A^{\nu,\ \ldots,c_i,\ldots}\left(\ldots,p_{i,\ldots}\right)|_{\{p_i\}=\left\{p_i'\right\}}. \tag{5}$$

As a result of charge conservation we have

$$A_g^{ext\ a,\ a_1,a_2,a_3,a_4}\left(l=k,\ k;\ldots\right)=$$
$$=ig\sum_i\sum_{c_i} f_{c_i}^{a\,a_i}\left(l_{2\ \nu}\left(k\frac{\partial}{\partial p_i}\right)A^{\nu,\ \ldots,c_i,\ldots}\left(\ldots,p_i,\ldots\right)|_{\{p_i\}=\left\{p_i'\right\}}\right). \tag{6}$$

Consequently, k - independent terms $O((k)^0)$ which may come from either A_g^{ext} or A_g^{int} are completely determined by the gauge-invariance requirement (for QED see [8]) $A_g\,(l = k, k, \ldots) = 0$ is given by

$$- ig \sum_i \sum_{c_i} f_{c_i}^{a\ a_i} l_{2\ \nu} \left(l \frac{\partial}{\partial p_i} \right) A^{\nu,\ \ldots,c_i,\ldots} (\ldots, p_i, \ldots)\ |_{\{p_i\}=\left\{p_i'\right\}} \tag{7}$$

Finally, in order to take into account the gauge invariance with respect to the second gluon in the process one should replace l_2 by l_2' where $l_2' p_2' = 0$ and a difference between l_2 and l_2' is order of $O\,(k)$. Then l_2' is reduced to $l_2^{\perp'}$ where $l_2' = B\,(l_2')\,p_2' +\ l_2^{\perp'}$ and $B\,(p_2') = 1,\ l_2^{\perp'} p_2' = 0$. The notation $\perp'$ emphasizes the transversality with respect to p_2'. In this way we obtain the expression for the total inelastic amplitude of the gluon production with small momentum k in terms of the elastic amplitude.

$$\begin{aligned} & A_g^{tot\ a,\ a_1,a_2,a_3,a_4}\,(l, k; p_1; l_2, p_2; p_3; p_4) = \\ & = +ig \textstyle\sum_i \sum_{c_i} f_{c_i}^{a\ a_i} \frac{lp_i}{kp_i} \left(l_{2\ \nu}^{\perp'} A^{\nu,\ \ldots,c_i,\ldots} (\ldots, p'_{i,\ldots}) + \right. \\ & \left. + \textstyle\sum_j \xi_j \frac{\partial}{\partial p_j} A^{\nu,\ \ldots,c_i,\ldots} (\ldots, p_{i,\ldots})\ |_{\{p_i\}=\left\{p_i'\right\}} \right) + \\ & +ig \textstyle\sum_i \sum_{c_i} f_{c_i}^{a\ a_i} \left(l_{2\ \nu}^{\perp'} D_i A^{\nu,\ \ldots,c_i,\ldots} (\ldots, p_i, \ldots) \right) |_{\{p_i\}=\left\{p_i'\right\}} + \\ & +ig \textstyle\sum_c f_c^{a\ a_2} \frac{\left(\left(l_2^{\perp'} l \right) k_{\ \nu} - l_{\ \nu} \left(l_2^{\perp'} k \right) \right)}{kp_2} A^{\nu,\ a_1,c,a_3,a_4} (\ldots, p_2', \ldots) \end{aligned} \tag{8}$$

where $D_i\,(k) = \frac{p_i}{kp_i} \left(k \frac{\partial}{\partial p_i} \right) - \frac{\partial}{\partial p_i}$. Then the property $k \cdot D_i\,(k) = 0$ and charge conservation expressed as $\sum_i \sum_{c_i} f_{c_i}^{a\ a_i} A^{\nu,\ \ldots,c_i,\ldots} (\ldots, p'_{i,\ldots}) = 0$ imply the required gauge-invariance properties $A_g^{tot\ a,\ a_1,a_2,a_3,a_4}\,(l = k,\ k; \ldots; \ldots) = 0$. The dependence on ξ_j is decorative and has to fall out in the final result.

The first term in the expression (8) involves terms of order of $O\,(\omega^{-1})$ while other terms are the corrections of order of $O\,(\omega^0)$ where ω is the boson frequency in the c.m. system. The last term in (8) can be treated as the contribution of a color anomalous magnetic moment.

The region of applicability of the Low formula can be extended [3] to the high-energy region where the momentum of the radiated gauge particle is

not small. Namely, it is sufficient to require that the emission of the gauge boson does not change the kinematics of the basic elastic process. Let us consider the basic elastic process of scattering in the Regge kinematics (see Fig.5) where $2p_1 \cdot p_2 = s \gg m_{char}^2$ and $(p_1 + p_4)^2 = t \sim m_{char}^2$, so that $\frac{t}{s} \to 0$. Going to the inelastic amplitude (see Fig.6), we introduce a Sudakov decomposition of the momentum of the radiated gluon

$$k = \alpha p_2 + \beta p_1 + k^{\perp} \tag{9}$$

where $k^{\perp} p_1 = k^{\perp} p_2 = 0,\ s\alpha\beta = k^{\perp\,2}$. Then the requirement to keep the kinematics of the basic process can be written as the following condition on the parameters $\alpha,\ \beta$

$$\alpha \ll 1 \quad , \quad \beta \ll 1 \quad , \quad \left| k^{\perp} \right| \ll m_{ch} \tag{10}$$

that is the fraction of energy which the gluon takes from the incoming particles is small and influence of the bremsstrahlung radiation onto the momentum transfer in the basic process is negligible.

$$\begin{gathered} s' - s = (p_1 + p_3)^2 - (p_1 + p_2)^2 \approx -(\alpha + \beta)\, s \ll s \\ t_2 - t_1 = 2qk^{\perp} \ll t \end{gathered} \tag{11}$$

We will look more carefully at this region below. The starting point of the further consideration is the elastic amplitude for the collision of three scalar and one vector particles (see Fig.5) in the Regge kinematics. In this case the polarization vector of the gluon can be decomposed into its longitudinal and transverse components with respect to momenta p_1 and p_2

$$l_2 = 2\frac{(l_2 p_1)}{s} p_2 + l_2^{\perp} \tag{12}$$

where $l_2^{\perp} p_1 = l_2^{\perp} p_2 = 0$. The elastic amplitude (in Fig.5) is factorized in its spin and unitary isospin indices according to a definite t-channel group representation

$$A_{2\to 1+g}{}^{a_4\, a_3}_{a_1\, a_2}(s,t) = \left(l_2^{\perp} q \right) A_{2\to 2}{}^{a_4\, a_3}_{a_1\, a_2}(s,t) \tag{13}$$

$$
\begin{gathered}
A_{2\to2}\,{}^{a_4\,a_3}_{a_1\,a_2}\,(s,t) = \sum_c \gamma^{a_4}_{a_1}(c)\,\gamma^{a_3}_{a_2}(c)\,A(s,t) = \\
\sum_c \gamma^{a_4}_{a_1}(c)\,\gamma^{a_3}_{a_2}(c)\,\beta(t)\left(\left(-\frac{S}{m_{ch}}\right)^{\alpha(t)} + \left(\frac{S}{m_{ch}}\right)^{\alpha(t)}\right)
\end{gathered}
\tag{14}
$$

where $q = p_1 + p_4 = -p_2 - p_3$ and a_i are the color indices of initial and final particles, c is an isotopic index of a reggeon, $\gamma^{a_4}_{a_1}(c)$ are Clebsch-Gordon coefficients. $A_{2\to2}\,{}^{a_4\,a_3}_{a_1\,a_2}\,(s,t)$ is the elastic amplitude for the collision of the scalar particles in the Regge kinematics. Below we use relations

$$
\sum_{c,d,e} \gamma^{a_3}_{a_2}(c)\left(f^{a\,a_4}_{d} + f^{a\,e}_{a_1}\right)\gamma^{d}_{e}(c) = \sum_{c,d} \gamma^{a_4}_{a_1}(c)\,T^{a}_{cd}\gamma^{a_3}_{a_2}(d)
$$

$$
\sum_{c,d,e} \gamma^{a_3}_{a_2}(c)\left(f^{a\,a_3}_{d} + f^{a\,e}_{a_2}\right)\gamma^{d}_{e}(c) = -\sum_{c,d} \gamma^{a_4}_{a_1}(c)\,T^{a}_{cd}\gamma^{a_3}_{a_2}(d)
$$

or

$$
(T_1 + T_4)\,\gamma(c) = T\gamma(c) \quad ; \quad \gamma(c)\,(T_2 + T_3) = -\gamma(c)\,T \tag{15}
$$

where the generators T_i in Eq. (15) act on the unitary spin indices of the i-th particle $(T_i)^a_c = f^{\ a}_{a_i\ c}$ and due to the gauge invariance

$$
(T_1 + T_2 + T_3 + T_4)\,A_{2\to2}(s,t) = 0, \tag{16}
$$

while the generator T acts on the unitary indices of the reggeon. For present purposes a representation of the $\xi_i(k)$ is defined by conservation of momenta and gauge invariance. We can obtain the $\xi_i(k)$ in the form $\xi_{1,2} = \frac{s_{1,2}+t_{1,2}}{s}p_{1,2} + \frac{k^{\perp}}{2}$, $\xi_3 = \xi_4 = 0$ or, in other words, the physically realizable set of on-mass-shell momenta $\{p'_i\}$ is given by

$$
\begin{gathered}
p'_1 = p_1\left(1 - \frac{s_1+t_1}{s}\right) + \frac{k^{\perp}}{2} \\
p'_2 = p_2\left(1 - \frac{s_2+t_2}{s}\right) + \frac{k^{\perp}}{2} \\
p'_3 = p_3 \quad , \quad p'_4 = p_4 \\
q = p'_1 + p'_4 = -p'_2 - p'_3
\end{gathered}
\tag{17}
$$

The contributions of the ξ -terms are of order of $O(k)$ in the bremsstrahlung amplitude and falls out. In this way for the kinematics

$$
\begin{gathered}
s_2 \sim s_{2'} \ll t_1 \sim t_2 \sim m_{ch}^2 \\
s_1 \sim s \gg m_{ch}^2
\end{gathered} \tag{18}
$$

we take into account the requirement (10) and continue the Low expression of the inelastic amplitude to this region,

$$
\begin{gathered}
A_g^{tot\ a,\ a_1,a_2,a_3,a_4}\left(l;k;p_1;l_2,p_2;p_3;p_4\right) = \\
I \\
ig\left\{-2\tfrac{lp_1}{s_1}\sum_{c,d}\gamma_{a_1}^{a_4}\left(c\right)T_{dc}^{a}\gamma_{a_1}^{a_3}\left(d\right)\left(l_2^{\perp}{}'q\right)A\left(s,t\right)\right. \\
II \\
-2\tfrac{lp_2}{s_2}\sum_{c,d}\gamma_{a_1}^{a_5}\left(c\right)\left(T_2\right)^{a\ d}\gamma_{d}^{a_3}\left(c\right)\left(l_2^{\perp}{}'q\right)A\left(s,t\right) \\
III \\
+2\tfrac{lp_3}{s_{2'}}\sum_{c,d}\gamma_{a_1}^{a_5}\left(c\right)\left(T_3\right)_{d}^{a_4}\gamma_{a_2}^{d}\left(c\right)\left(l_2^{\perp}{}'q\right)A\left(s,t\right) \\
IV \\
+\left[-\left(\tfrac{lp_1}{s_1}\left(l_2^{\perp}{}'k\right)+\tfrac{1}{2}\left(l_2^{\perp}{}'l\right)\right)+\left(l_2^{\perp}{}'q\right)\left(lB_1\right)\tfrac{\partial}{\partial t}\right]\times \\
\times\sum_{c,d}\gamma_{a_1}^{a_4}\left(c\right)T_{dc}^{a}\gamma_{a_1}^{a_3}\left(d\right)\left(l_2^{\perp}{}'q\right)A\left(s,t\right) \\
V \\
+\left[\left(\tfrac{lp_2}{s_2}\left(l_2^{\perp}{}'k\right)+\tfrac{1}{2}\left(l_2^{\perp}{}'l\right)\right)+2\tfrac{1}{s_2}\left(\left(l\left(l_2^{\perp}{}'k\right)-\left(l_2^{\perp}{}'l\right)k\right)_{\nu}q^{\nu}\right)\right. \\
\left.-\left(l_2^{\perp}{}'q\right)\left(lB_2\right)\tfrac{\partial}{\partial t}\right]\times \\
\times\sum_{c,d}\gamma_{a_1}^{a_4}\left(c\right)\left(T_2\right)^{a\ d}\gamma_{d}^{a_3}\left(c\right)\left(l_2^{\perp}{}'q\right)A\left(s,t\right) \\
VI \\
+\left[\left(-\tfrac{lp_3}{s_{2'}}\left(l_2^{\perp}{}'k\right)+\tfrac{1}{2}\left(l_2^{\perp}{}'l_4\right)\right)-\left(l_2^{\perp}{}'q\right)\left(l_4B_{2'}\right)\tfrac{\partial}{\partial t}\right]\times \\
\times\sum_{c,d}\gamma_{a_1}^{a_4}\left(c\right)\left(T_3\right)_{d}^{a}\gamma_{a_2}^{d}\left(c\right)\left(l_2^{\perp}{}'q\right)A\left(s,t\right)
\end{gathered} \tag{19}
$$

where

$$
\begin{array}{ccc}
B_1 = (t_1 - t_2)\frac{p_1}{s_2} - \frac{q_1+q_2}{2} & , & kB_1 = 0 \\
B_2 = (t_1 - t_2)\frac{p_2}{s_2} - \frac{q_1+q_2}{2} & , & kB_2 = 0 \\
B_{2'} = -(t_1 - t_2)\frac{p_3}{s_{2'}} - \frac{q_1+q_2}{2} & , & kB_{2'} = 0 \\
\multicolumn{3}{c}{t_i = q_i^2 \ , \ q_1 = p_1 + p_4 \ , \ q_2 = p_2 + p_3}
\end{array}
\tag{20}
$$

In order to derive (19) we have used that in the kinematics in (18) one has

$$
\begin{gathered}
D_1(k)\, A(s,t) = B_1 A(s,t) \approx D_4(k)\, A(s,t) \\
D_2(k)\, A(s,t) = B_2 A(s,t) \quad D_{2'}(k)\, A(s,t) = B_{2'} A(s,t) \\
B_1 \approx B_{1'} = B_5 \qquad s_2 B_2 = s_{2'} B_{2'}
\end{gathered}
\tag{21}
$$

Last three terms IV, V and VI in Eq.(19) are corrections of order of $\left(k^{\perp}\right)^0$ to the first three terms which are of the order $\left(k^{\perp}\right)^{-1}$. Unlike the scalar particle collisions, Eq.(19) contains the additional terms $\sim \left(k^{\perp}\right)^0$ related to the non-zero spin particles. In the s_2 -channel the corrections $\sim \left(\frac{lp_2}{s_2}\left(l_2^{\perp\prime}k\right) + \frac{1}{2}\left(l_2^{\perp\prime}l\right)\right) + 2\frac{1}{s_2}\left(\left(l\left(l_2^{\perp\prime}k\right) - \left(l_2^{\perp\prime}l\right)k\right)_\nu q^\nu\right)$ related to the anomalous color quadrupole electric and magnetic dipole moments of the gluon. In the s_1 and $s_{2'}$ -channels the corrections of order of $\left(k^{\perp}\right)^0$ like terms $\sim \left(-\frac{lp_i}{s_i}\left(l_2^{\perp\prime}k\right) + \frac{1}{2}l_2^{\perp\prime}l\right)$ can be treated as induced vertices for two color scalar particles with two vectors.

Let us consider the expression in (19) in the limit $s_2 \to \infty$ that is in the multi-Regge kinematics

$$
\begin{gathered}
s_1 \gg m_{ch}^2 \quad , \quad s_2 \gg m_{ch}^2 \quad , \quad \frac{s_1 s_2}{s} = \overrightarrow{k}_{\perp}^2 \ll m_{ch}^2 \\
t_1, t_2 \sim m_{ch}^2
\end{gathered}
\tag{22}
$$

In this case for $k_\perp \ll m_{ch}$ the amplitude in (19) takes the form

$$
A_{2\to 2+g}^{a,\ a_1,a_2,a_3,a_4}(s, s_1, s_2, t_1, t_2)\Big|_{k_\perp \to 0} = \ g^3 \ \times
$$

$$
l_4^\mu \left[2\left(\frac{p_2}{s_2} - \frac{p_1}{s_1}\right)\left(l_2^{\perp\prime}q\right) - l_2^{\perp\prime} - \left(\frac{p_2}{s_2} + \frac{p_1}{s_1}\right)\left(l_2^{\perp\prime}k\right) + (B_1 + B_2)\frac{\partial}{\partial t}\right]_\mu \times \tag{23}
$$

$$\times \sum_{i,j} \gamma_{a_1}^{a_4}(i)\, T_{ji}^{a} \gamma_{a_1}^{a_3}(j) \left(l_2^{\perp\prime} q\right) A(s,t) \quad .$$

The first term in (23) is the leading pole Gribov term of order $\frac{1}{\left|k^{\perp}\right|}$ and other are the corrections of order $\left|k^{\perp}\right|^{0}$. In this formula contribution of the magnetic moment is suppressed as $O\left(\frac{\overrightarrow{k^{\perp}}^{2}}{s_2}\right)$. But there are terms $\sim -l_2^{\perp\prime} - \left(\frac{p_2}{s_2} + \frac{p_1}{s_1}\right)\left(l_2^{\perp\prime} k\right)$ and this expression does not agree with the amplitude factorization which must be in the multi-Regge kinematics. In order to correct the situation it is necessary to take into account some additional contributions. Below we represent the correct amplitude and explain the mechanism of appearance of this contributions using the bosonic string as guide.

3 Contribution of intermediate excited states in the multi-Regge kinematics.

In this section we will try to take into account the contribution of intermediate excited states in the $s_1, s_{1'}, s_2$ and $s_{2'}$ channels in addition to the Low formula above, which describes the contribution of the ground state in these channels. For the present case the ground states are considered to be scalar in the $s_1, s_{1'}, s_{2'}$ channels and vector in the s_2 channel. It is important to emphasize that the contribution from excited intermediate states which is small ($\sim \omega$) due to gauge invariance [3]. However, in the regions of the large s_i it turns out is of order $\left|k^{\perp}\right|^{0}$, that is of the same order as of the ordinary correction of the ground states to the Low formula. The necessity of the inelastic contributions follows from the correct analytic behavior for the scattering amplitude [5]. Taking into account the excited states we have to relax the condition $s_i \sim 0$. The simplest way is to consider the production of massless particles with small transverse momenta in the multi-Regge kinematics in such a model which would allow us to take into account excited states.

As discussed earlier, the leading terms $\sim O\left(\frac{1}{\left|k^{\perp}\right|}\right)$ in the Low formula describe correctly the gluon bremsstrahlung amplitude in regions (10) and

(22). As shown in Ref.[5] the correction related to excited states is fixed by general analyticity in the channels s_1, and s_2 up to a constant which cannot be obtained from general consideration. Calculation of this constant was illustrated by an example of string bosonic model, when the gauge particle was produced in scattering of four scalar particles [4] and also of four massless vector particles [6]. Since we would like to establish the dependence of the gluon bremsstrahlung amplitude on the nature of the external particles then it is interesting to consider the mixed case of particles with different spin. In the multi-Regge kinematics the calculation is rather simple, furthermore, it also allows us to control further calculations in other region.

In what follows we conform to the bosonic string model to illustrate the further consideration of the problem. The first step is to consider the elastic string amplitude of the scattering of the scalar particles and a vector particle in the Regge kinematics. In this process tachyons which are ground states in the string play a role of the scalar particles while the vector particle is an excited state of the open string. In the Born approximation we have (see Fig.5)

$$A_{2\to 1+g}(s,t,u) \ \sim \ \int \prod_{i=1}^{4} dx_i \prod_{i<j} |x_i - x_j|^{-2\alpha' p_i p_j} V^1(x), \tag{24}$$

where p_1, p_2, p_3 and p_4 are the momenta of external on-shell particles $p_i^2 = p_{i_0}^2 - \vec{p}_i^{\,2} = -\frac{1}{\alpha'}$ $(i \neq 2)$ and $p_2^2 = 0$. We choose independent scalar invariants $t = (p_1 + p_4)^2$ and $s = 2p_1p_2$. Other scalar products are linear combinations of the s and t ; α' is a slope of the Regge trajectory $\alpha(t) = 1 + \alpha' t$. The vertex $V^1(x)$ depends on polarization vector l_2^μ, momenta p_i^μ $(i \neq 2)$ and Koba-Nielsen variables x_i.

$$V^1(x) = \sum_{m\neq 2} \frac{l_2 p_m}{x_m - x_2} \tag{25}$$

It is not difficult to verify invariance of Eq.(25) with respect to gauge transformation $l_i(p_i) \to l_i(p_i) + cp_i$, taking into account the identity

$$p_2 \sum_{i\neq 2} \frac{(p_i l_2)}{x_i - x_2} \ = \ \frac{\partial}{\partial x_2} \ \sum_{i<j} 2p_ip_j \ln |x_i - x_j|. \tag{26}$$

The vertex $V^1(x)$ and expression (24) are invariant with respect to the

Möbius group

$$x_i' \;=\; \frac{(ax_i+b)}{(cx_i+d)}, \qquad ad-bc \;=\; 1 \tag{27}$$

and we have to take into account the factor $|x_{13}x_{34}x_{41}|$ which gives a volume of this group to obtain the correct expression of the amplitude

$$A^4(s,t) \;\sim \int \prod_{i=1}^{4} dx_i \left|\frac{x_{21}x_{34}}{x_{31}x_{42}}\right|^{-\alpha' s} \left|\frac{x_{41}x_{32}}{x_{31}x_{42}}\right|^{-\alpha' t} V^1(x)|x_{13}x_{34}x_{41}|, \tag{28}$$

where $x_{ij} = x_i - x_j$. Thanks to (27) , we can fix three variables of integration to be

$$x_1 = \;0, \qquad x_2 = \;x, \qquad x_3 = \;1, \qquad x_4 = \;\infty. \tag{29}$$

The Yang-Mills color group is introduced simply by multiplying every string diagrams by the corresponding Chan-Paton factor [8] according to the order of external particles. In such a case, different regions of integration are combined in the amplitude, so that every one enters with its corresponding weight

$$\begin{aligned} C_{a_1a_2a_3a_4} \;&=\; \tfrac{1}{2}\, tr\,(\gamma_{a_1}\gamma_{a_2}\gamma_{a_3}\gamma_{a_4})\,, \quad tr\,\gamma_i\gamma_j \;=\; \delta_{ij} \\ C_{a_1a_2\ldots a_n} \;&=\; (-1)^n C_{a_na_{n-1}\ldots a_1}\,, \end{aligned} \tag{30}$$

where γ_i are matrices of the fundamental representation of color $SU(N)$ group.

Thus, after the normalization of the amplitude we get

$$\begin{aligned} A_{2\to 1+g}(s,t) \;&=\; g^2 \left(C_{a_2a_1a_3a_4}\textstyle\int_{-\infty}^{0} + C_{a_1a_2a_3a_4}\int_0^1 + C_{a_1a_3a_2a_4}\int_1^{\infty}\right)|x|^{-\alpha(s)-1} \\ &\times \; |1-x|^{-\alpha(t)-1}\, V^1(x)\, dx. \end{aligned} \tag{31}$$

In the case of the string model we have $m_{ch} \sim \sqrt{\frac{1}{\alpha'}}$ and in the Regge asymptotics

$$s \;\gg\; \frac{1}{\alpha'}, \qquad t \;\sim\; \frac{1}{\alpha'}, \tag{32}$$

the essential region of integration is $|1-x| = \epsilon \sim 1/\alpha' s$ and the first term in Eq.(31) turns to be asymptotically small.

In the multi-Regge kinematics it is convenient to expand the polarization vector into longitudinal and transverse parts with respect to the vectors p_1

and p_2 (see(12)). The main asymptotic contribution in the amplitude takes the form

$$A_{2\to 1+g}^{a_1a_2a_3a_4}(s,t) \;=\; g^2\left(l_2^{\perp}q\right)\left(C_{a_1a_2a_3a_4}\int_0^{\infty}+C_{a_1a_3a_2a_4}\int_{-\infty}^{0}\right)\,e^{-\alpha' s\epsilon}\epsilon^{-\alpha' t}\frac{d\epsilon}{\epsilon^2}, \tag{33}$$

where $q=p_1+p_4=-p_2-p_3$ (see Fig.5).

After the integration and the analytical continuation of this expression we obtain the elastic amplitude which has the factorized form

$$A_{2\to 1+g}^{a_1a_2a_3a_4}(s,t) \;=\left(l_2^{\perp}q\right)A_{2\to 2}^{a_1a_2a_3a_4}(s,t), \tag{34}$$

where

$$A_{2\to 2}^{a_1a_2a_3a_4}(s,t) \;=\; g^2\left((-\alpha' s)^{\alpha(t)}C_{a_1a_2a_3a_4}+(\alpha' s)^{\alpha(t)}C_{a_1a_3a_2a_4}\right)\,\Gamma(-\alpha(t)). \tag{35}$$

$A_{2\to 2}(s,t)$ is the elastic scattering tachyon amplitude [4], $\Gamma(x)$ is the Gamma-function.

Now we consider the gluon production amplitude in the Koba-Nielsen representation (see Fig.7)

$$A_{2\to 1+g+g}(s,s_1,s_2,t_1,t_2)\sim\int\prod_{i=1}^{5}dx_i\prod_{i<j}|x_i-x_j|^{-2\alpha' p_ip_j}V^2(x_i). \tag{36}$$

Here momenta p_i of external particles are on mass shell $p_2^2=p_4^2=0, p_1^2=p_3^2=p_5^2=-\frac{1}{\alpha'}$. We choose as independent the following scalar invariants

$$\begin{gathered} s=2p_1p_2,\quad s_1=-2p_1p_4,\quad s_2=-2p_2p_4,\\ t_1=2p_1p_5-\tfrac{2}{\alpha'},\quad t_2=2p_2p_3-\tfrac{1}{\alpha'}. \end{gathered} \tag{37}$$

All other invariants can be expressed in terms of the above ones as

$$\begin{gathered} 2p_1p_3 \;=\; -s+s_1-t_1,\qquad 2p_2p_5 \;=\; -s+s_2-t_2-\frac{1}{\alpha'}\\ 2p_3p_4\equiv s_{2'}=s_2+t_1-t_2,\quad 2p_3p_5=s-s_1-s_2-\frac{1}{\alpha'},\\ 2p_4p_5\equiv s_{1'}=s_1-t_1+t_2. \end{gathered} \tag{38}$$

The vertex $V^2(x_i)$ of the emission of the two vector particles is given by [6, 9].

$$V^2\left(x_i\right) = -\frac{l_2 l_4}{\left(x_2 - x_4\right)^2} + 2\alpha' \sum_2^5 \sum_4^5 \tag{39}$$

where

$$\sum_m^5 = \sum_{l \neq m}^5 \frac{l_m p_l}{x_l - x_m} \ .$$

We also add the factor $|x_{15} x_{12} x_{52}|$ to cancel the infinite volume of the Möbius group and fix three Koba-Nielsen variables

$$x_1 = 0,\ x_2 = 1,\ x_3 = x,\ x_4 = y,\ x_5 = \infty$$

and then rewrite the expression of the amplitude as follows

$$\begin{aligned} A_{2\to 1+g+g}(s, s_1, s_2, t_1, t_2) \sim \int dxdy \left|\tfrac{1}{x}\right|^{-\alpha' s} \left|\tfrac{x}{y}\right|^{-\alpha' s_1} \left|\tfrac{x-y}{1-y}\right|^{-\alpha' s_2} \times \\ \times \left|\tfrac{x-y}{x}\right|^{-\alpha' t_1} \left|\tfrac{1-x}{x-y}\right|^{-\alpha' t_2} \left|\tfrac{x}{(1-x)^2}\right| \times \\ \times \left(-\tfrac{l_2 l_4}{y^2} + 2\alpha' l_2^\mu \left(p_2 + \tfrac{p_3}{x} + \tfrac{p_4}{y}\right)_\mu \ l_4^\nu \left(-\tfrac{p_1}{y} + \tfrac{p_2}{1-y} + \tfrac{p_3}{x-y}\right)_\nu\right) \end{aligned} \tag{40}$$

The fourth particle are considered as the radiated one so we denote $k = p_4$ and $l = l_4$. Similarly to the cases of scattering of tachyons [4] or gluons [6] we first estimate the expression in (40) in the limit $\overrightarrow{k_\perp}^2 \to 0$ and then take the limit $\overrightarrow{k_\perp} \to 0$.

In the multi-Regge kinematics in (22) the main contribution gives the region

$$\left.\begin{array}{l} \frac{x}{y} - 1 \equiv \epsilon_1 \sim \frac{1}{\alpha' s_1} \to 0 \\ \\ \frac{x-y}{1-y} \equiv \epsilon_2 \sim \frac{1}{\alpha' s_2} \to 0 \end{array}\right\} \quad \Rightarrow \quad \begin{array}{c} x \approx 1 + \epsilon_1 \epsilon_2 \\ \\ y \approx 1 - \epsilon_1 \end{array} \ . \tag{41}$$

Thus, for the new variables the representation of the amplitude in (40) is to be divided into four pieces which correspond to positive or negative ϵ_1, ϵ_2 . In each piece the values of s, s_1 and s_2 have to be chosen positive or

negative in such a way, that integrals converge. After integration one has to continue them to the physical region of the s-channel (or another channel) where s, s_1 and s_2 take positive values. In addition we take into account their color Chan-Paton factors (30) (see Fig.8) and introduce new variables $x_1,\ x_2$

$$x_1 = \alpha' s_1 \epsilon_1 \quad , \quad x_2 = \alpha' s_2 \epsilon_2 \ .$$

After extracting the factors $(\alpha' s_1)^{\alpha(t_1)}$ and $(\alpha' s_2)^{\alpha(t_2)}$, the remaining integral depends on the invariants s, s_1 and s_2 only via the combination $\Lambda = \frac{s}{\alpha' s_1 s_2} = -\frac{1}{\alpha' \overrightarrow{k_\perp}^2}$. For small $\overrightarrow{k_\perp}^2 \to 0$ and therefore for large $\Lambda \to \infty$ the main contribution comes from the regions $x_1 \sim \frac{1}{\Lambda}\ ,\ x_2 \sim 1$ and $x_2 \sim \frac{1}{\Lambda}\ ,\ x_1 \sim 1$. In doing so, we obtain the inelastic amplitude (see Fig.7) in the multi-Regge kinematics, which is factorized in its spin indices

$$A^{a_1a_2a_3a_4a_5}_{2\to 1+g+g}(s, s_1, s_2, t_1, t_2)\Big|_{\overrightarrow{k_\perp}^2 \to 0} = \left(l_2^\perp q_2\right)\ A^{a_1a_2a_3a_4a_5}_{2\to 2+g}(s, s_1, s_2, t_1, t_2)\Big|_{\overrightarrow{k_\perp}^2 \to 0} \tag{42}$$

where $A_{2\to 2+g}(s, s_1, s_2, t_1, t_2)|_{\overrightarrow{k_\perp}^2 \to 0}$ is the gluon production amplitude in the case of the collision involving only tachyons [4]

$$\begin{aligned}
A^{a_1a_2a_3a_4a_5}_{2\to 2+g}(s, s_1, s_2, t_1, t_2)\Big|_{\overrightarrow{k_\perp}^2 \to 0} = \ \alpha' g^3 \ \times \\
\Big\{C_{a_1a_2a_3a_4a_5}\left((-\alpha' s)^{\alpha(t_1)}(-\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)}J_2 + (-\alpha' s)^{\alpha(t_2)}(-\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)}J_1\right) \\
+\ C_{a_1a_4a_2a_3a_5}\left((-\alpha' s)^{\alpha(t_1)}(\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)}J_2 + (-\alpha' s)^{\alpha(t_2)}(\alpha' s_2)^{\alpha(t_1)-\alpha(t_2)}J_1\right) \\
+\ C_{a_1a_3a_2a_4a_5}\left((\alpha' s)^{\alpha(t_1)}(\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)}J_2 + (\alpha' s)^{\alpha(t_2)}(-\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)}J_1\right) \\
+\ C_{a_1a_4a_3a_2a_5}\left((\alpha' s)^{\alpha(t_1)}(\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)}J_2 + (\alpha' s)^{\alpha(t_2)}(\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)}J_1\right)\Big\}
\end{aligned} \tag{43}$$

where

$$\begin{aligned}
J_1 &= \Gamma(-\alpha(t_2))\Gamma(\alpha(t_2)-\alpha(t_1))B^1_\mu l^\mu \ . \\
J_2 &= \Gamma(-\alpha(t_1))\Gamma(\alpha(t_1)-\alpha(t_2))B^2_\mu l^\mu .
\end{aligned} \tag{44}$$

The amplitude in (42) has the correct analytical structure and the simultaneous singularities in the s_1 and s_2 channels are absent. Consider now the result in (44) in the region

$$\left|\overrightarrow{k_\perp}\right| \ll m_{ch} \quad . \tag{45}$$

In this case $m_{ch} \sim q_i \sim \frac{1}{\sqrt{\alpha'}}$ and in the physical region we have simultaneously $\frac{1}{\Lambda} \to 0$ and $\alpha'(t_1 - t_2) = \alpha'(q_1 + q_2)(q_1 - q_2) \sim \alpha'(qk_\perp) \to 0$. We want to obtain an expression for the total inelastic amplitude of the gluon production with small momentum $k_\perp$ in terms of elastic amplitude (34). For this purpose, as discussed earlier, we have to express the momenta and the polarization vector of the inelastic amplitude in terms of the corresponding values in the elastic amplitude. But we have to be careful to keep the gauge invariance and the conservation of momenta. In other words we take the physically realizable set of on-mass-shell momenta $\{p'_i\}$ (see Eq.(17)) and the polarization vector l'_2 for the elastic amplitude

$$l'_2 \;=\; l_2 - \tfrac{l_2^\perp k}{s}\, p_1 + \tfrac{l_2 p_1}{s}\, k^\perp$$

or, in terms of the transverse component

$$l_2^\perp \;=\; l_2^{\perp\prime} - \tfrac{(l_2{}^{\perp\prime} k)}{s}\, p'_1 - \tfrac{(l_2^{\perp\prime} k)}{s}\, p'_2$$

Expanding Eq.(43) in $k^\perp \to 0$ around l'_2 and q, we obtain for the gluon bremsstrahlung amplitude

$$\begin{aligned}
&A^{a_1a_2a_3a_4a_5}_{2\to 2+g}(s, s_1, s_2, t_1, t_2)\Big|_{k_\perp \to 0} \;=\; g^3 \;\times \\
&\Bigg\{ l^\mu \left[2\left(\frac{p_2}{s_2} - \frac{p_1}{s_1}\right)\left(l_2^{\perp\prime} q\right) - \left(\frac{p_2}{s_2} - \frac{p_1}{s_1}\right)\left(l_2^{\perp\prime} k\right) + (B_1 + B_2)\left(l_2'^{\perp\prime} q\right)\frac{\partial}{\partial t}\right]_\mu \times \\
&\quad\times\; \Gamma(-\alpha(t))\Big((C_{a_1a_2a_3a_4a_5} + C_{a_1a_4a_2a_3a_5})(-\alpha' s)^{\alpha(t)} + \\
&\quad\quad + (C_{a_1a_3a_2a_4a_5} + C_{a_1a_4a_3a_2a_5})(\alpha' s)^{\alpha(t)} \Big) \\
&+2\alpha'(l_4 B_1)\left[\left(\psi(1) + \ln\tfrac{1}{-\alpha' s_1}\right) \times \right. \\
&\times \left(l_2^{\perp\prime} q\right)\left(C_{a_1a_2a_3a_4a_5}(-\alpha' s)^{\alpha(t)} + C_{a_1a_3a_2a_4a_5}(\alpha' s)^{\alpha(t)}\right)\Gamma\left(-\alpha\left(t\right)\right)
\end{aligned} \tag{46}$$

$$
\begin{aligned}
&+ \left(\psi(1) + \ln \tfrac{1}{\alpha' s_1}\right) \times \\
&\times \left(l_2^{\perp\prime} q\right) \left(C_{a_1 a_4 a_2 a_3 a_5} (-\alpha' s)^{\alpha(t)} + C_{a_1 a_4 a_3 a_2 a_5} (\alpha' s)^{\alpha(t)}\right) \Gamma\left(-\alpha\left(t\right)\right)\Big] \\
&+2\alpha'(lB_2) \Big[\left(\psi(1) + \ln \tfrac{1}{-\alpha' s_2}\right) \times \\
&\times \left(l_2^{\perp\prime} q\right) \left(C_{a_1 a_2 a_3 a_4 a_5} (-\alpha' s)^{\alpha(t)} + C_{a_1 a_4 a_3 a_2 a_5} \left(\alpha' s\right)^{\alpha(t)}\right) \Gamma\left(-\alpha\left(t\right)\right) \\
&+ \left(\psi(1) + \ln \tfrac{1}{\alpha' s_2}\right) \times \\
&\times \left(l_2^{\perp\prime} q\right) \left(C_{a_1 a_4 a_2 a_3 a_5} (-\alpha' s)^{\alpha(t)} + C_{a_1 a_3 a_2 a_4 a_5} (\alpha' s)^{\alpha(t)}\right) \Gamma\left(-\alpha\left(t\right)\right)\Big]\Big\}
\end{aligned}
$$

where $\psi\left(x\right) = \frac{\frac{\partial}{\partial x}\Gamma(x)}{\Gamma(x)}$ and B_1, B_2 are defined by (20). The first term in (46) is the non-Abelian generalization of Gribov's pole term of order of $O\left(\frac{1}{k_\perp}\right)$ and some corrections of order $O\left(k_\perp^0\right)$. The contribution from the color anomalous magnetic moment is suppressed as $O\left(\frac{k_\perp^2}{s_2}\right)$ in this kinematics. The last terms describe the additional $O\left(k_\perp^0\right)$ contribution of the intermediate excited states in the s_1 and s_2 channels. Eq.(46) can be considered as the dispersion representation in the invariants s_i with the subtraction at $s_i = 0$ [5]. The subtraction constants which provide the agreement with low energy behavior (where the contribution of the intermediate excited states dies out as $s_i \approx \omega m_{ch} \to 0$) are equal in all channels. They are $\psi(1)$.

On the other hand, we could expect the difference in the values of this constants in s_1 and s_2 channels because of the presence of the vector particle in the s_2 channel. Due to the external vector particle which is really the first excited state of the open string the subtraction point at $s_2 = 0$ means the vector state in the channel. It lies above the tachyon point in the spectrum of states so this additional excited tachyon contribution like $lB_2\left(\frac{1}{\alpha' s_2 - 1} + 1\right)\left(l_2^{\perp\prime} q\right)\Gamma\left(-\alpha\left(t\right)\right)\left(C_{a_1 a_2 a_3 a_4 a_5}(-\alpha' s)^{\alpha(t)} + C_{a_1 a_4 a_3 a_2 a_5}\left(\alpha' s\right)^{\alpha(t)}\right)$ could be in the fragmentation regions where $s_2 \sim \frac{1}{\alpha'}$ and could change the subtraction constant like $\psi(1) - 1$. But in this case in the multi-Regge limit where $s_2 \to \infty$ we would have another tensor and color structure $\sim lB_2\left(l_2^{\perp\prime} q\right)$ than in (46) and such a simple interpretation of the difference of the subtraction constants is wrong.

Look at the expression (46) once more. It is in full agreement with the amplitude factorization. The difference from the multi-Regge limit of the Low

expression in (23) implies that the tensor structure $\sim -\left(\frac{p_2}{s_2}-\frac{p_1}{s_1}\right)\left(l_2^{\perp\prime}k\right)$ is in the first term in (46) instead of terms $\sim -l_2^{\perp\prime}-\left(\frac{p_2}{s_2}+\frac{p_1}{s_1}\right)\left(l_2^{\perp\prime}k\right)$ in (23) plus the contribution of the intermediate excited states which is the same as for the gluon production amplitude in the case of the scalar particles collision.

The explanation of the amplitude factorization can be due to the fact that the contribution of some intermediate excited states has another tensor and color structure which allows it to be mixed with the standard Low corrections and to restore the correct factorization amplitude behavior. In order to justify this interpretation we should consider the dispersion relations in detail. This is possible in the fragmentation region which is considered in the next section.

4 Contribution of intermediate excited states in the fragmentation region.

In this section we select the region of scalar invariants which is called the fragmentation region (see Fig.9).

$$s_1 \sim s \gg m_{ch}^2 \quad , \quad s_2 \sim s_{2'} \sim m_{ch}^2 \tag{47}$$

We will use the string model as well as earlier where $m_{ch}^2=\frac{1}{\alpha'}$. The starting point in this section is the expression in (40). We will follow here the estimation of integrals in Ref.[5]. In the kinematics (47) the essential region of integration in (40) is

$$\left.\begin{array}{c} 1-x\equiv\epsilon\sim\frac{1}{\alpha' s}\to 0 \\ \\ \frac{x-y}{1-y}\equiv z\sim 1\to 1 \end{array}\right\} \quad \Rightarrow \quad \left\{\begin{array}{c} x\approx 1-\epsilon \\ \\ y\approx 1-\frac{\epsilon}{1-z} \end{array}\right. \tag{48}$$

and the amplitude $A_{2\to 1+g+g}(s,s_1,s_2,t_1,t_2)$ is given by

$$A_{2\to 1+g+g}(s,s_1,s_2,t_1,t_2)\ \sim$$

$$\sim \int \frac{d\epsilon}{\epsilon^2}\exp\left[-\alpha' s\epsilon-\alpha' s_1\epsilon\frac{z}{1-z}\right]|z|^{-\alpha' s_2}\left|\epsilon\frac{z}{1-z}\right|^{-\alpha' t_1}\left|\frac{1-z}{z}\right|^{-\alpha' t_2}$$
$$\times\, l^{\perp}_{2\,\nu}\cdot\left(l+2\alpha'\left(q_2\frac{z}{(1-z)^2}+q_1\frac{1}{1-z}\right)\right)^{\nu}\times \tag{49}$$
$$\times l_{\ \mu}\cdot\left(p_1\epsilon+p_2\frac{(1-z)^2}{z}+q_2\frac{1-z}{z}\right)^{\mu}$$

Now, according to the kinematics (10) we can take

$$s_1 \quad \ll \quad s. \tag{50}$$

Then in this asymptotics there are two essential integration regions in formula in (49)

$$1.)\quad \alpha' s_1\epsilon\frac{z}{z-1}\sim 1 \tag{51}$$

$$2.)\qquad \frac{z}{z-1}\sim 1\ . \tag{52}$$

The amplitude receives two sorts of contributions which come from regions (51) and (52), respectively.

$$A=A_1+A_2 \tag{53}$$

where A_1 and A_2 possess different analytic properties. We estimate the contribution of the regions when $\alpha' s\sim\alpha' s_1\sim\infty$ and then consider the limit $\frac{s_1}{s}\to 0$.

It is worth noting that the Chan-Paton factors appear with their signs which reflect a number of twists for tree amplitudes [8]. Physical states of definite mass are eigenstates of the twist operator with eigenvalue $(-1)^N$ where $N=-1$ for the tachyons and $N=0$ for the massless vector particle.

In the region (51) we take

$$\left.\begin{array}{c}\alpha' s\epsilon = x_1\\[2ex] \alpha' s_1\epsilon\frac{z}{z-1}=x_2\end{array}\right\}\quad\Rightarrow\quad\left\{\begin{array}{c}\epsilon=\frac{x_1}{\alpha' s}\\[2ex] z=\frac{1}{\alpha' s_1\left(1+\frac{x_1}{x_2}\right)}\end{array}\right. \quad . \tag{54}$$

The contribution A_1 is reduced to already well-known analytic and Chan-Paton structure from the multi-Regge computation (compare with (43) and see Fig.7 for the Chan-Paton factors)

$$
\begin{aligned}
A_1 = \quad & g^3 \times \\
\times \ \Big\{ & C_{a_1a_2a_3a_4a_5}(-\alpha' s)^{\alpha(t_2)}(-\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)} \\
+ \ & C_{a_1a_4a_2a_3a_5}(-\alpha' s)^{\alpha(t_2)}(\alpha' s_2)^{\alpha(t_1)-\alpha(t_2)} \\
+ \ & C_{a_1a_3a_2a_4a_5}(\alpha' s)^{\alpha(t_2)}(-\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)} \\
+ \ & C_{a_1a_4a_3a_2a_5}(\alpha' s)^{\alpha(t_2)}(\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)} \Big\} \cdot J_1\left(\frac{s_1}{s}, s_2, t_1, t_2\right)
\end{aligned}
\tag{55}
$$

where

$$
\begin{aligned}
J_1\left(\tfrac{s_1}{s}, s_2, t_1, t_2\right) = & -\int\int_0^\infty dx_1 dx_2 x_1^{-2-\alpha' t_2} e^{-x_1} x_2^{-1+\alpha'(t_2-t_1)} e^{-x_2} \tfrac{x_1}{x_2}\left(1-\tfrac{s_1 x_1}{s x_2}\right) \\
\times \quad & l_{2\,\nu}^{\perp} \cdot \left(l\tfrac{s_1}{s} + 2\alpha'\left(q_2\left(\tfrac{x_2}{x_1}\right)^2 + q_1\tfrac{s_1 x_1}{s x_2}\right)\right)^\nu \\
& l_{\ \mu} \cdot \left(\tfrac{x_1}{\alpha' s_1} p_1 + \tfrac{s_1 x_1}{s x_2} p_2 + \tfrac{x_1}{x_2} q_2\right)^\mu
\end{aligned}
\quad .
\tag{56}
$$

From the second region in (52)

$$
\left\{
\begin{aligned}
\epsilon &= \tfrac{1}{\alpha' s} x_1 \\
z &= z
\end{aligned}
\right.
\tag{57}
$$

taking into account the Chan-Paton factors (see Fig.10) we obtain

$$
\begin{aligned}
& A_2 = g^3 \quad \times \\
& \times \ \Big\{\Big[C_{a_1a_2a_3a_4a_5}\left(-\alpha' s\right)^{\alpha(t_1)} + C_{a_1a_4a_3a_2a_5}\left(\alpha' s\right)^{\alpha(t_1)}\Big]\,\Phi_1\left(\tfrac{s_1}{s}, s_2, t_1, t_2\right) \\
& - \Big[C_{a_1a_4a_2a_3a_5}\left(-\alpha' s\right)^{\alpha(t_1)} + C_{a_1a_3a_2a_4a_5}\left(\alpha' s\right)^{\alpha(t_1)}\Big]\,\Phi_2\left(\tfrac{s_1}{s}, s_2, t_1, t_2\right) \\
& - \Big[C_{a_1a_2a_4a_3a_5}\left(-\alpha' s\right)^{\alpha(t_1)} + C_{a_1a_3a_4a_2a_5}\left(\alpha' s\right)^{\alpha(t_1)}\Big]\,\Phi_3\left(\tfrac{s_1}{s}, s_2, t_1, t_2\right)\Big\}
\end{aligned}
\tag{58}
$$

where

$$\Phi_1\left(\frac{s_1}{s},s_2,t_1,t_2\right)=-\int_0^\infty dx_1\int_0^1 dz x_1^{-2-\alpha' t_1}e^{-x_1-\frac{s_1}{s}x_1\frac{z}{1-z}}\times$$
$$\times z^{-\alpha' s_2}\left(\frac{z}{1-z}\right)^{-\alpha' t_1}V_2(x,z)$$
$$\Phi_2\left(\frac{s_1}{s},s_2,t_1,t_2\right)=-\int_0^\infty dx_1\int_1^\infty dz x_1^{-2-\alpha' t_1}e^{-x_1-\frac{s_1}{s}x_1\frac{z}{1-z}}\times$$
$$\times z^{-\alpha' s_2}\left(-\frac{z}{1-z}\right)^{-\alpha' t_1}V_2(x,z) \tag{59}$$
$$\Phi_3\left(\frac{s_1}{s},s_2,t_1,t_2\right)=-\int_0^\infty dx_1\int_{-\infty}^0 dz x_1^{-2-\alpha' t_1}e^{-x_1-\frac{s_1}{s}x_1\frac{z}{1-z}}\times$$
$$\times(-z)^{-\alpha' s_2}\left(-\frac{z}{1-z}\right)^{-\alpha' t_1}V_2(x,z)$$
$$V_2(x,z)=l^{\perp}_{2\,\nu}\left(l+2\alpha'\left(q_2\frac{z}{(1-z)^2}+q_1\frac{1}{1-z}\right)\right)^{\nu}$$
$$l_\mu\left(p_1\frac{x_1}{\alpha' s}+p_2\frac{(1-z)^2}{z}+q_2\frac{1-z}{z}\right)^{\mu}.$$

In the limit $\frac{s_1}{s}\to 0$ we have

$$J_1\left(\frac{s_1}{s},s_2,t_1,t_2\right)\Big|_{\frac{s_1}{s}\to 0}=2\alpha'\left(l_2^{\perp}q_2\right)(l_4B_1)\cdot K_0$$
$$\Phi_1\left(\frac{s_1}{s},s_2,t_1,t_2\right)\Big|_{\frac{s_1}{s}\to 0}=\alpha' A_{423}(s_2,s_{2'},t_1,t_2)\cdot K_1$$
$$\Phi_2\left(\frac{s_1}{s},s_2,t_1,t_2\right)\Big|_{\frac{s_1}{s}\to 0}=\alpha' A_{423}(s_2,s_{2'},t_1,t_2)\cdot K_2 \tag{60}$$
$$\Phi_3\left(\frac{s_1}{s}(s_2,t_1,t_2\right)\Big|_{\frac{s_1}{s}\to 0}=\alpha' A_{423}(s_2,s_{2'},t_1,t_2)\cdot K_3$$

where

$$
\begin{aligned}
&K_0 = \Gamma\left(-\alpha\left(t_2\right)\right)\Gamma\left(\alpha\left(t_2\right)-\alpha\left(t_1\right)\right)\\
&K_1 = -\Gamma\left(-\alpha\left(t_1\right)\right)\tfrac{\Gamma(1-\alpha' s_{2'})\Gamma(\alpha(t_1)-\alpha(t_2))}{\Gamma(2-\alpha' s_2)}\\
&K_2 = -\Gamma\left(-\alpha\left(t_1\right)\right)\tfrac{\Gamma(-1+\alpha' s_{2'})\Gamma(\alpha(t_1)-\alpha(t_2))}{\Gamma(\alpha' s_{2'})}\\
&K_3 = \Gamma\left(-\alpha\left(t_1\right)\right)\tfrac{\Gamma(-1+\alpha' s_2)\Gamma(1-\alpha' s_{2'})}{\Gamma(1+\alpha'(t_2-t_1))}
\end{aligned}
\tag{61}
$$

and

$$
\begin{aligned}
&A_{423}\left(s_2, s_{2'}, t_1, t_2\right) = \left(t_1-t_2\right)\left(l_2^{\perp} l\right) + 2\left(l_2^{\perp} k\right)\left(l q_2\right) + \\
&+2\alpha' s_2\left(l B_2\right)\left(l_2^{\perp} q_2\right) - 2\left(l B_{2'}\right)\left(l_2^{\perp} q_1\right)\left(1+\alpha'\left(t_1-t_2\right)\right),
\end{aligned}
\tag{62}
$$

the vectors B_1, B_2 and $B_{2'}$ were given earlier (see (20)). In the asymptotics (13) we have $t_2 \to t_1$ ($q_2 \to q_1$) and can take physically realizable set (17) of on-shell momenta $\{p_i'\}$. In order to keep the gauge invariance we also have to make the change $l_2 \to l_2'$ ($l_2' p_2' = 0$)

$$
\begin{aligned}
&l_2^{\perp} q_2 = l_2^{\perp\prime} q + \tfrac{l_2^{\perp\prime} k}{2}\\
&l_2^{\perp} q_1 = l_2^{\perp\prime} q - \tfrac{l_2^{\perp\prime} k}{2}.
\end{aligned}
\tag{63}
$$

It should be stressed that in the physical region where $s_2 \geq 0$ and $s_{2'} \leq 0$ the expression $\Phi_2\left(\frac{s_1}{s}, s_2, t_1, t_2\right)\Big|_{\frac{s_1}{s}\to 0}$ has poles only in s_2 while $\Phi_1\left(\frac{s_1}{s}, s_2, t_1, t_2\right)\Big|_{\frac{s_1}{s}\to 0}$ has poles only in $s_{2'}$ (see Eqs.(60) and (61)). It reflects the fact that these expressions describe the contribution of the different channels in the amplitude. One of them describes the contribution of the emission of vector and scalar particles ($s_{2'}$ -channel) and another describes the contribution of the emission of two vector (s_2 -channel).

In this way Eq.(60) can be simplified in accordance with different regions of s_i (the results of the simplification of the corresponding functions are represented in the appendix). Substituting Eqs.(69), (70), (73) and (76) into Eqs.(56), (58) we obtain for the gluon production amplitude in the region $\alpha' s_2 \sim \alpha' s_{2'} \sim 0$ in the Gribov limit (see (10)) :

$$A\left(s,s_1,s_2,t_1,t_2\right)\big|_{\substack{\alpha' s,\ \alpha' s_1\to\infty\\ \alpha' s_2\sim 0}} =$$

$$I$$

$$= 2g^3\left\{\left[-2\tfrac{lp_1}{s_1}l_2^{\perp\prime}q-\tfrac{1}{2}l_2^{\perp\prime}l-\tfrac{lp_1}{s_1}l_2^{\perp\prime}k+\left(lB_1\right)\left(l_2^{\perp\prime}q\right)\tfrac{\partial}{\partial t}\right]\times\right.$$

$$\times\Gamma\left(-\alpha\left(t\right)\right)\Big(\left(-\alpha' s\right)^{\alpha(t)}\tfrac{1}{2}\left(C_{a_1a_2a_3a_4a_5}+C_{a_1a_4a_2a_3a_5}\right)+$$

$$+\left(\alpha' s\right)^{\alpha(t)}\tfrac{1}{2}\left(C_{a_1a_3a_2a_4a_5}+C_{a_1a_4a_3a_2a_5}\right)\Big)+$$

$$II$$

$$+\alpha'\left(lB_1\right)\Big[\Big(\psi\left(1\right)+\log\tfrac{1}{-\alpha' s_1}\Big)\left(l_2^{\perp\prime}q\right)\Gamma\left(-\alpha\left(t\right)\right)\times$$

$$\times\Big(\left(-\alpha' s\right)^{\alpha(t)}C_{a_1a_2a_3a_4a_5}+\left(\alpha' s\right)^{\alpha(t)}C_{a_1a_3a_2a_4a_5}\Big)+$$

$$+\Big(\psi\left(1\right)+\log\tfrac{1}{\alpha' s_1}\Big)\left(l_2^{\perp\prime}q\right)\Gamma\left(-\alpha\left(t\right)\right)\times$$

$$\times\Big(\left(-\alpha' s\right)^{\alpha(t)}C_{a_1a_4a_2a_3a_5}+\left(\alpha' s\right)^{\alpha(t)}C_{a_1a_4a_3a_2a_5}\Big)\Big]+$$

$$III \tag{64}$$

$$+\left[-2\frac{lp_3}{s_{2'}}l_2^{\perp\prime}q-\frac{1}{2}l_2^{\perp\prime}l+\frac{lp_3}{s_{2'}}l_2^{\perp\prime}k+\left(lB_{2'}\right)\left(l_2^{\perp\prime}q\right)\frac{\partial}{\partial t}\right]\times$$

$$\times\Gamma\left(-\alpha\left(t\right)\right)\Big(\left(-\alpha' s\right)^{\alpha(t)}\tfrac{1}{2}\left(C_{a_1a_2a_3a_4a_5}+C_{a_1a_2a_4a_3a_5}\right)+$$

$$+\left(\alpha' s\right)^{\alpha(t)}\tfrac{1}{2}\left(C_{a_1a_4a_3a_2a_5}+C_{a_1a_3a_4a_2a_5}\right)\Big)+$$

$$IV$$

$$+\left[2\frac{lp_2}{s_2}l_2^{\perp\prime}q-\frac{1}{2}l_2^{\perp\prime}l-\frac{lp_2}{s_2}l_2^{\perp\prime}k-\frac{\left(t_1-t_2\right)l_2^{\perp\prime}l+2\left(l_2^{\perp\prime}k\right)lq^{\perp}}{s_2}+\right.$$

$$\left.+\left(lB_2\right)\left(l_2^{\prime\perp\prime}q\right)\frac{\partial}{\partial t}\right]\times$$

$$\times\Gamma\left(-\alpha\left(t\right)\right)\left(\left(-\alpha' s\right)^{\alpha(t)}\frac{1}{2}\left(C_{a_1a_4a_2a_3a_5}-C_{a_1a_2a_4a_3a_5}\right)+\right.$$

$$\left.\left.+\left(\alpha' s\right)^{\alpha(t)}\frac{1}{2}\left(C_{a_1a_3a_2a_4a_5}-C_{a_1a_3a_4a_2a_5}\right)\right)\right\}$$

Eq.(64) allows us to single out the contribution of the nonexcited states in the s_2 and $s_{2'}$ channel. This expression can be compared with Eq.(19). In Eq. (64) the I and II terms are related to the emission of the gauge particle from lines 1 and 5, while terms III and IV are associated with the emission from lines 2 and 3. The II term gives us the contribution of excited intermediate states in the s_1-channel (it is additional to (19)). Among the III and IV terms the contributions of such states are absent and we see that the structure of these terms is in full agreement with the Low formula. There are only pole leading terms $\sim O\left(\frac{1}{k_\perp}\right)$ and the corrections $\sim O\left(k_\perp^0\right)$ coming from them (compare with terms IV and VI in Eq.(19)). And it is worth noting that terms I and III contain the terms $\sim -\frac{1}{2}l_2^{\perp\prime}l-\frac{lp_i}{s_i}l_2^{\perp\prime}k$ which are associated with the induced vertex of interactions of two color scalar particles with two vector particles. Term IV contains the contribution $\sim -\frac{1}{2}l_2^{\perp\prime}l-\frac{lp_2}{s_2}l_2^{\perp\prime}k-\frac{(t_1-t_2)l_2^{\perp\prime}l+2\left(l_2^{\perp\prime}k\right)lq^{\perp}}{s_2}$ which can be interpreted as the contribution of the color quadrupole electric moment and of the color magnetic dipole moment (see term IV in Eq.(19)). Thus the amplitude in (64) except the contribution of the excited intermediate states in the s_1-channel for bremsstrahlung radiation from scalar particles (which coincides with [5]) is in full agreement with the generalization of the Low expressions in (19).

The calculation of the bremsstrahlung amplitude in the fragmentation region $\alpha' s_2\sim\alpha' s_{2'}\sim 1$ in the Gribov limit can be completed if we substitute the expressions $(69)\,,(71)\,,(74)$ and (77) into Eq.(56) and Eq.(58).

$$A\left(s,s_1,s_2,t_1,t_2\right)\Big|_{\substack{\alpha' s,\ \alpha' s_1\to\infty\\ \alpha' s_2\sim\alpha' s_2\sim 1}}=$$

$$I$$

$$=2g^3\left\{\left[-2\frac{lp_1}{s_1}l_2^{\perp\prime}q-\frac{1}{2}l_2^{\perp\prime}l-\frac{lp_1}{s_1}l_2^{\perp\prime}k+\left(lB_1\right)\left(l_2^{\perp\prime}q\right)\frac{\partial}{\partial t}\right]\times\right.$$

$$\times\Gamma\left(-\alpha\left(t\right)\right)\Big(\left(-\alpha's\right)^{\alpha(t)}\tfrac{1}{2}\left(C_{a_1a_2a_3a_4a_5}+C_{a_1a_4a_2a_3a_5}\right)+$$

$$\left(\alpha's\right)^{\alpha(t)}\tfrac{1}{2}\left(C_{a_1a_3a_2a_4a_5}+C_{a_1a_4a_3a_2a_5}\right)\Big)+$$

$$II$$

$$+\alpha'\left(lB_1\right)\Big[\Big(\psi\left(1\right)+\log\tfrac{1}{-\alpha's_1}\Big)\Big(l_2^{\perp\prime}q\Big)\Gamma\left(-\alpha\left(t\right)\right)\times$$

$$\times\Big(\left(-\alpha's\right)^{\alpha(t)}C_{a_1a_2a_3a_4a_5}+\left(\alpha's\right)^{\alpha(t)}C_{a_1a_3a_2a_4a_5}\Big)+$$

$$+\Big(\psi\left(1\right)+\log\tfrac{1}{\alpha's_1}\Big)\Big(l_2^{\perp\prime}q\Big)\Gamma\left(-\alpha\left(t\right)\right)\times$$

$$\times\Big(\left(-\alpha's\right)^{\alpha(t)}C_{a_1a_4a_2a_3a_5}+\left(\alpha's\right)^{\alpha(t)}C_{a_1a_4a_3a_2a_5}\Big)\Big]+$$

$$III$$

$$+\left[-2\frac{lp_3}{s_{2'}}l_2^{\perp\prime}q-\frac{1}{2}l_2^{\perp\prime}l+\frac{lp_3}{s_{2'}}l_2^{\perp\prime}k+\left(lB_{2'}\right)\Big(l_2^{\perp\prime}q\Big)\frac{\partial}{\partial t}\right]\times$$

$$\times\Gamma\left(-\alpha\left(t\right)\right)\Big(\left(-\alpha's\right)^{\alpha(t)}C_{a_1a_2a_3a_4a_5}+\left(\alpha's\right)^{\alpha(t)}C_{a_1a_4a_3a_2a_5}\Big)+$$

$$IV \tag{65}$$

$$+\left[2\left(\frac{1}{\alpha's_{2'}-1}+1\right)\left(\frac{1}{2}l_2^{\perp\prime}l-\frac{lp_3}{s_{2'}}l_2^{\perp\prime}k\right)+\right.$$

$$\left.+\,2\alpha'\left(lB_{2'}\right)\left(\psi\left(1\right)-\psi\left(1-\alpha's_{2'}\right)\right)\Big(l_2^{\perp\prime}q\Big)\right]\Gamma\left(-\alpha\left(t\right)\right)\times$$

$$\times\Big(\left(-\alpha's\right)^{\alpha(t)}C_{a_1a_2a_3a_4a_5}+\left(\alpha's\right)^{\alpha(t)}C_{a_1a_4a_3a_2a_5}\Big)+$$

$$V$$

$$+\left[2\frac{lp_2}{s_2}l_2^{\perp\prime}q-\frac{1}{2}l_2^{\perp\prime}l-\frac{lp_2}{s_2}l_2^{\perp\prime}k+\left(lB_2\right)\Big(l_2^{\perp\prime}q\Big)\frac{\partial}{\partial t}\right]\times$$

$$\times\Gamma\left(-\alpha\left(t\right)\right)\Big(\left(-\alpha's\right)^{\alpha(t)}C_{a_1a_4a_2a_3a_5}+\left(\alpha's\right)^{\alpha(t)}C_{a_1a_3a_2a_4a_5}\Big)+$$

$$VI$$

$$+\left[2\left(\frac{1}{\alpha's_2-1}+1\right)\left(\frac{1}{2}l_2^{\perp\prime}l+\frac{lp_2}{s_2}l_2^{\perp\prime}k\right)+\right.$$

$$+ 2\alpha' (lB_2) (\psi (1) - \psi (1 + \alpha' s_2)) \left(l_2^{\perp\prime} q \right) \Big] \Gamma (-\alpha (t)) \times$$

$$\times \left((-\alpha' s)^{\alpha(t)} C_{a_1 a_2 a_3 a_4 a_5} + (\alpha' s)^{\alpha(t)} C_{a_1 a_4 a_3 a_2 a_5} \right) -$$

$$VII$$

$$-2 \left(\frac{lp_2}{s_2} + \frac{lp_3}{s_{2'}} \right) \left(l_2^{\perp\prime} q \right) \frac{\pi \alpha' s_2}{\sin \pi \alpha' s_2} \times$$

$$\times \Gamma (-\alpha (t)) \left((-\alpha' s)^{\alpha(t)} C_{a_1 a_2 a_4 a_3 a_5} + (\alpha' s)^{\alpha(t)} C_{a_1 a_3 a_4 a_2 a_5} \right) \Big\}$$

Terms I and II describe the emission of gauge particle from the scalar lines 1 and 5. They coincide with the same terms in the multi-Regge kinematics since $s_1 \gg \frac{1}{\alpha'}$ (see Eq.(64)). Term I in Eq.(65) is a non-Abelian generalization of Low's formula [2] (compare with Eq.(19)). The second term in Eq.(65) represents the additional contribution of excited intermediate states in the s_1 channel. Terms III, IV and V, VI describe the emission from scalar line 3 and vector line 2 respectively. Also terms III and V are the non-Abelian generalizations of Low's formula while the IV and VI ones give us the additional contribution of the excited intermediate states in the $s_{2'}$ and s_2 channels in the fragmentation region where $\alpha' s_2 \sim \alpha' s_{2'} \sim 1$. As far as term VII is concerned it represents the u-channel contribution of the elastic amplitude. To interpret the results we used that

$$\psi (1) - \psi (1 + \alpha' s_2) = \sum_{n=1}^{\infty} \left(\frac{1}{\alpha' s_2 + n} - \frac{1}{n} \right) \tag{66}$$

$$\psi (1) - \psi (1 - \alpha' s_{2'}) = \sum_{n=1}^{\infty} \left(\frac{1}{-\alpha' s_{2'} + n} - \frac{1}{n} \right) \tag{67}$$

and also

$$\lim_{\alpha' s_i \to \infty} \psi (1 \pm \alpha' s_i) \to \log (\pm \alpha' s_i) \, . \tag{68}$$

Due to relations (66),(67) and (68) it is obvious that the inelastic amplitude in the fragmentation region has infinitely many poles in the $s_{2'}$ and s_2 channels. In this way formulae (46) and (65) can be considered as dispersion representations in s_i invariants with subtraction at $s_i = 0$. Inspecting (65) we note that in general the residues of these poles are equal to each other. These pieces with the equal residues represent the contributions of the same intermediate excited states as for the bremsstrahlung amplitude in the case

of the scalar particle collision (see [5]). But in the $s_{2'}$ channel (see term IV in Eq.(65)) we have the additional pole contribution at $\alpha' s_{2'} = 1$ ($m^2 = 0$) which is associated with the additional excited intermediate vector state and the residue in this pole is different from the contributions of other excited intermediate states in this channel in comparison with the gluon production amplitude for the scalar collisions (see [5]). In the s_2 channel we have also the additional pole contribution at $\alpha' s_2 = 1$ which can be interpreted as the contribution of the excited intermediate tachyon state ($m^2 = -\frac{1}{\alpha'}$) and the residue in this pole is different from the other one. As seen from Eq.(65) and (66) the subtraction constant of the dispersion representation is given by the set of the subtraction constants for each pole contribution of the intermediate excited states in the corresponding s_2 or $s_{2'}$ channel. Its value is determined by two additive constants $\psi(1)$ and -1 but with different tensor factors. In doing so, in the s_2 channel the $\psi(1)$ gives us the contribution to the subtraction constant from those excited states which lie in the mass spectrum above the external vector state while -1 is related to the excited intermediate tachyon which lies below external gluon in the mass spectrum. In the $s_{2'}$ channel both constants $\psi(1)$ and -1 are associated with the excited intermediate states in the mass spectrum which lie above the external tachyon.

Finally, from Eq.(65) in the limit $\alpha' s_2 \sim \alpha' s_{2'} \sim \infty$ we return to Eq.(46) for each of the t-channel invariant amplitude. As a check on this result one can also obtain the same one if one substitutes expressions $(69), (72), (75)$ and (78) into Eqs.(56) and (58).

In the limit $s_2 \to \infty$ the contributions of the subtraction constants of the additional intermediate excited tachyon and vector states are provided with such a color and tensor structure which allows them to be mixed with contributions $\sim \left|k^{\perp}\right|^0$ of the nonexcited states. In doing so, we observe that the terms $\sim l_2^{\perp\prime} l$ which are in contradiction with the amplitude factorization are cancelled by the contributions of the additional intermediate excited states in the s_2 and $s_{2'}$ channels, more exactly, due to their subtraction constants. The correct amplitude factorization behavior is restored. For the field theory it means that it is necessary to take into account additionally to pole diagrams the contributions of the other one in which the contribution of the scalar and vector intermediate states is in the corresponding channels. Since in the field theory we take into account the only pole diagrams it is

the reason why in the multi-Regge kinematics the structure of expression in (23) for the field theory and Eq.(46) differ from each other. And we see that the corresponding subtraction constant for the additional intermediate vector and scalar states in the $s_{2'}$ and s_2 channel is used to restore the correct amplitude factorization while the subtraction constant of the other excited intermediate states is kept. In this way the additive constant really does not depend on the nature of external particles from which the gluon is radiated.

5 Summary

Thus, in this paper we have shown that in the high energy collision of scalar and vector particles the production amplitude of the massless gauge particles with the small transverse momentum does not coincide with the non-Abelian generalization of the Low formulae. These formulae are in contradiction with the correct amplitude factorization. The mechanism of the restoration of the amplitude factorization implies that the additional intermediate scalar and vector states are excited in the s_2 and $s_{2'}$ channels respectively. The other intermediate states are in full agreement with the same ones as for the gluon production amplitude for the scalar collision [5]. In the Regge model these terms in the amplitudes have non-pole singularities in the invariants s_i (see also Eq.(46) in the multi-Regge kinematics. The additive constant (which characterizes the mass spectrum of the intermediate states, the regime of the Regge behavior and which fits the amplitude behavior at high energy of the radiated gluon to the amplitude behavior at low energy of the radiated gluon) does not depend on the nature of the external particles i.e. whether scalar or vector particles are the sources of the gluon radiation. Also we have demonstrated that the production amplitude of the massless gauge particles with small transverse momentum in the fragmentation region of initial particles has nontrivial analytical and other interesting properties in these invariants s_i (for example, the equality of the residues of the poles is independent of the nature of the external particles). The Low-Gribov theorem and its generalization are used in the field theory to obtain evolution equations for scattering amplitudes in the Regge kinematics [10]. In turn, the evolution equations are the starting point of the effective two-dimensional theory [11]. It can be equivalent to ordinary QCD. The explanation of the structure of terms entering into the amplitude allows us to obtain important information

for building the whole effective field theory. In particular, from Eq.(65) we can obtain vertices like 'reggeon and two or three particles'.

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A

Here we represent the result of calculations of the functions $J_1\left(\frac{s_1}{s}, s_2, t_1, t_2\right)$ and $\Phi_i\left(\frac{s_1}{s}, s_2, t_1, t_2\right)$ $i = 1, 2, 3$ in the different kinematical regions.

$$\alpha' s_1 \sim \infty$$

$$J_1|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \alpha'\Gamma\left(-\alpha\left(t\right)\right)\left[-\frac{2(lB_1)\left(l_2^{\perp\prime}q\right)}{2\alpha' x} - \frac{(lB_1)\left(l_2^{\perp\prime}k\right)}{2\alpha' x} + \right. \\ \left. + \left(lB_1\right)\left(l_2^{\perp\prime}q\right)\chi\left(t\right)\right] \tag{69}$$

$$\alpha' s_{2'} \sim 0$$

$$\Phi_1|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \alpha'\Gamma\left(-\alpha\left(t\right)\right)\left[-\frac{1}{\alpha'}\left(l_2^{\perp\prime}l - \frac{lp_3}{s_{2'}}\left(l_2^{\perp\prime}k\right)\right) - \frac{(lq)\left(l_2^{\perp\prime}k\right)}{2\alpha' x} + \right. \\ \left. + \left(lB_{2'}\right)\left(l_2^{\perp\prime}q\right)\left(\frac{1}{\alpha' x} - \psi\left(\alpha\left(t\right)\right)\right)\right] \tag{70}$$

$$\alpha' s_{2'} \sim 1$$

$$\Phi_1|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \alpha'\Gamma\left(-\alpha\left(t\right)\right)\left[\frac{\left(l_2^{\perp\prime} l - \frac{lp_3}{s_{2'}}\left(l_2^{\perp\prime} k\right)\right)\left(\alpha' s_{2'}+1\right)}{\alpha'\left(\alpha' s_{2'}-1\right)} - \right. \tag{71}$$

$$\left. -\frac{(lq)\left(l_2^{\perp\prime} k\right)}{2\alpha' x} + 2\left(lB_{2'}\right)\left(l_2^{\perp\prime} q\right)\left(\frac{1}{2\alpha' x} + \chi\left(t\right) - \psi\left(1-\alpha' s_{2'}\right)\right)\right]$$

$$\alpha' s_{2'} \sim \infty; \qquad s_2 \approx s_{2'} \ , \ B_2 \approx B_{2'}$$

$$\Phi_1|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \alpha'\Gamma\left(-\alpha\left(t\right)\right)\left[\frac{lp_2}{\alpha' s_2} l_2^{\perp\prime} k - \frac{(lq)\left(l_2^{\perp\prime} k\right)}{2\alpha' x} + \right. \tag{72}$$

$$\left. +2\left(lB_{2'}\right)\left(l_2^{\perp\prime} q\right)\left(\frac{1}{2\alpha' x} + \chi\left(t\right) - \log\left(-\alpha' s_{2'}\right)\right)\right]$$

$$\alpha' s_2 \sim 0$$

$$\Phi_2|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = -\alpha'\Gamma\left(-\alpha\left(t\right)\right)\left[-\frac{1}{\alpha'}\left(l_2^{\perp\prime} l - \frac{lp_2}{s_2}\left(l_2^{\perp\prime} k\right)\right) - \frac{(lq)\left(l_2^{\perp\prime} k\right)}{2\alpha' x} + \right. \tag{73}$$

$$\left. +2\frac{x\left(l_2^{\perp\prime} l\right) + (lq)\left(l_2^{\perp\prime} k\right)}{\alpha' s_2} + \left(lB_2\right)\left(l_2^{\perp\prime} q\right)\left(\frac{1}{\alpha' x} - \psi\left(\alpha\left(t\right)\right)\right)\right]$$

$$\alpha' s_2 \sim 1$$

$$\Phi_2|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \alpha'\Gamma\left(-\alpha\left(t\right)\right)\left[\frac{\left(l_2^{\perp\prime} l - \frac{lp_2}{s_2}\left(l_2^{\perp\prime} k\right)\right)\left(\alpha' s_2+1\right)}{\alpha'\left(\alpha' s_2-1\right)} - \right. \tag{74}$$

$$\left. -\frac{(lq)\left(l_2^{\perp\prime} k\right)}{2\alpha' x} + 2\left(lB_2\right)\left(l_2^{\perp\prime} q\right)\left(\frac{1}{2\alpha' x} + \chi\left(t\right) - \psi\left(1+\alpha' s_2\right)\right)\right]$$

$$\alpha' s_2 \sim \infty; \qquad s_2 \approx s_{2'} \ , \ B_2 \approx B_{2'}$$

$$\Phi_2|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \alpha' \Gamma\left(-\alpha\left(t\right)\right)\left[\frac{lp_2}{\alpha' s_2} l_2^{\perp\prime} k - \frac{(lq)\left(l_2^{\perp\prime} k\right)}{2\alpha' x} + \right. \tag{75}$$

$$\left. +2\left(lB_2\right)\left(l_2^{\perp\prime} q\right)\left(\frac{1}{2\alpha' x} + \chi\left(t\right) - \log\left(\alpha' s_{2'}\right)\right)\right]$$

$$\alpha' s_2 \sim \alpha' s_{2'} \sim 0$$

$$\Phi_3|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \Gamma\left(-\alpha\left(t\right)\right)\left[\left(\frac{lp_2}{s_2} + \frac{lp_3}{s_{2'}}\right) \times \right. \tag{76}$$

$$\left. \times \left(2 l_2^{\prime\perp\prime} q - l_2^{\perp\prime} k - 2\alpha' x \left(l_2^{\prime\perp\prime} q\right)\psi\left(\alpha\left(t\right)\right)\right) - 2\frac{x\left(l_2^{\perp\prime} l\right) + (lq)\left(l_2^{\perp\prime} k\right)}{s_2}\right]$$

$$\alpha' s_2 \sim \alpha' s_{2'} \sim 1$$

$$\Phi_3|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = \Gamma\left(-\alpha\left(t\right)\right)\left(\frac{lp_2}{s_2} + \frac{lp_3}{s_{2'}}\right)\left(l_2^{\perp\prime} q\right)\frac{2\pi\alpha' s_2}{\sin \pi \alpha' s_2} \tag{77}$$

$$\alpha' s_2 \sim \alpha' s_{2'} \sim \infty$$

$$\Phi_3|_{\substack{\frac{s_1}{s}\to\infty \\ |k^\perp|^2 \ll \frac{1}{\alpha'}}} = 0 \quad (\textit{go to } \infty \textit{ along real axis in imaginary plane}) \tag{78}$$

where

$$t_1 - t_2 \equiv x = -k^\perp q$$

$$s_{2'} = s_2 + t_1 - t_2 = s_2 + x$$

$$\chi\left(t\right) = \psi\left(1\right) - \tfrac{1}{2}\psi\left(-\alpha\left(t\right)\right)$$

$$\psi\left(x\right) = \frac{\Gamma'(x)}{\Gamma(x)} \quad \psi\left(1+x\right) = \psi\left(x\right) + \frac{1}{x}$$

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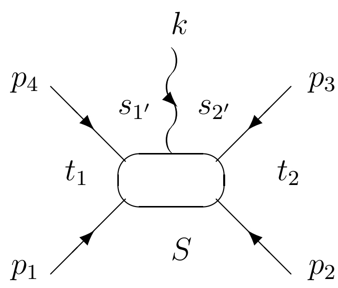

Figure 1: The five-particle amplitude.

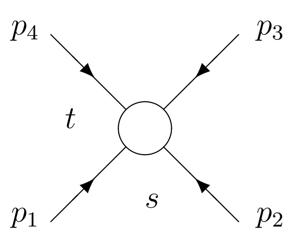

Figure 2: The elastic scalar particle amplitude.

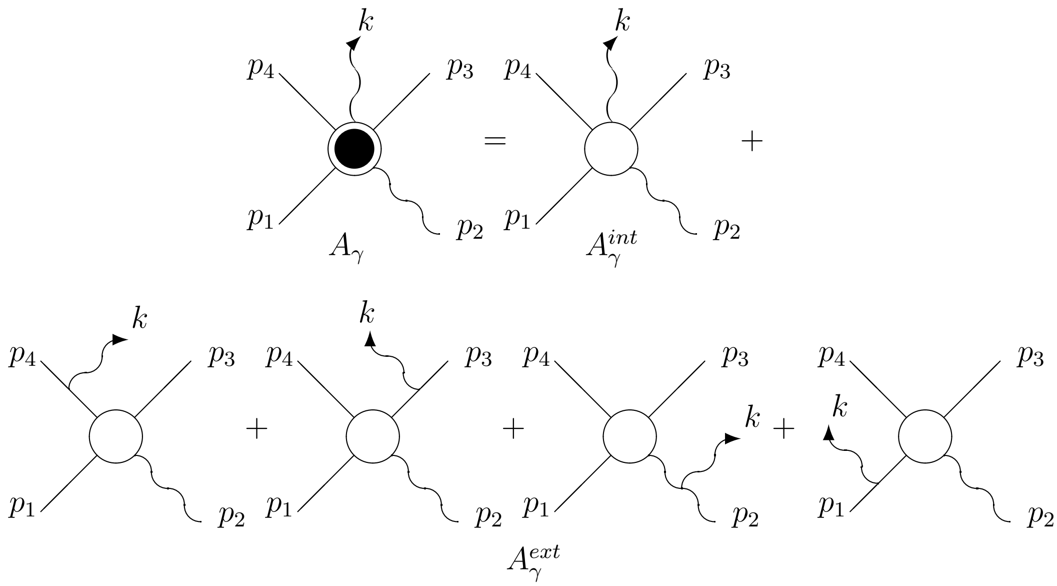

Figure 3: The amplitude can be divided into an external and internal part.

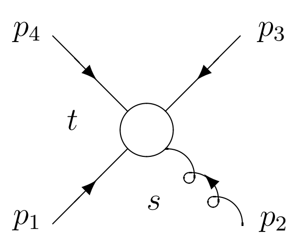

Figure 4: The basic elastic amplitude with one vector and three scalar particles.

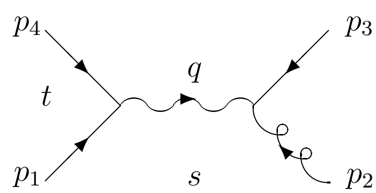

Figure 5: The basic elastic process of scattering in the Regge kinematics.

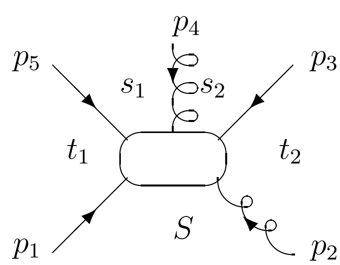

Figure 6: The inelastic amplitude.

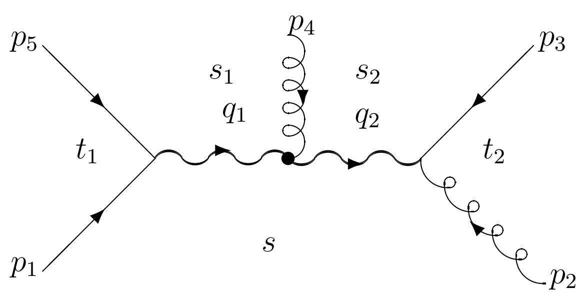

Figure 7: The gluon production amplitude in the multi-Regge kinematics.

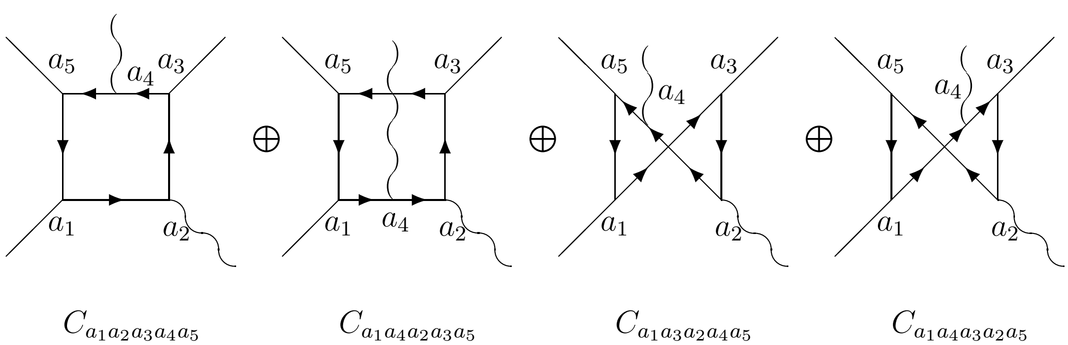

Figure 8: Chan-Paton factors which contribute in the multi-Regge kinematics.

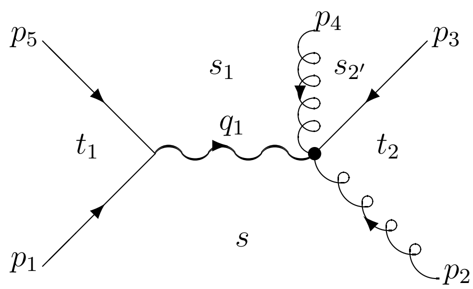

Figure 9: Massless particle production amplitude in the fragmentation region.

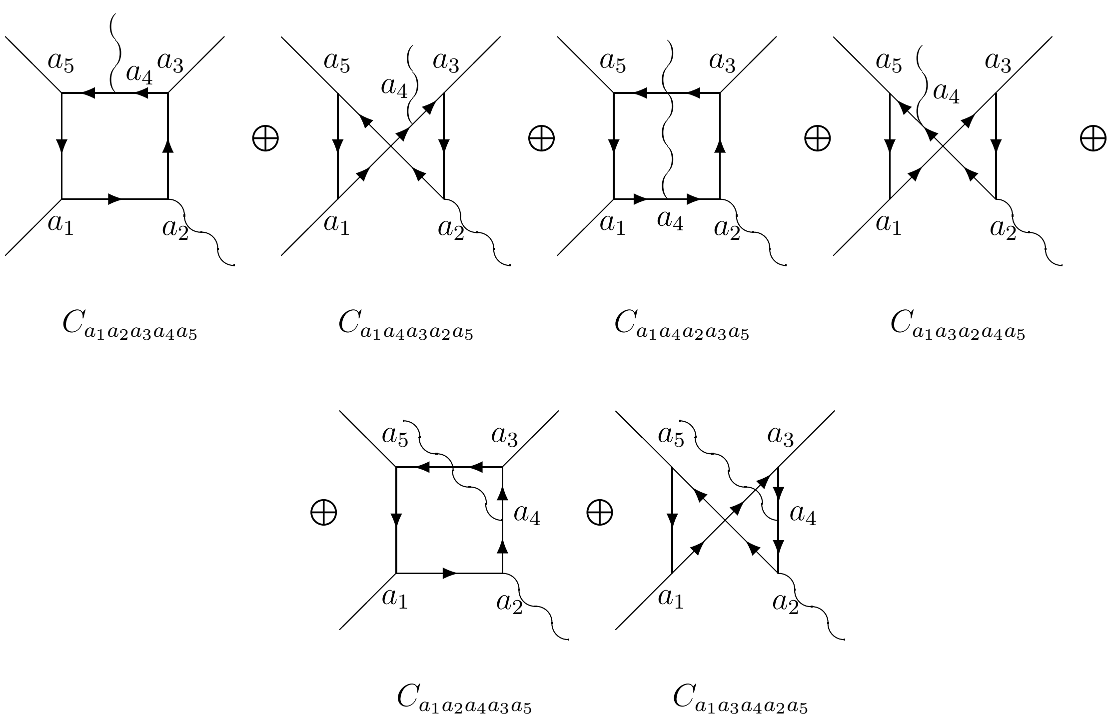

Figure 10: Chan-Paton factors which contribute into the A_2 piece of the gluon production amplitude in the fragmentation region.